Now, it is a defect of [natural] languages that expressions are possible within them, which, in their grammatical form, seemingly determined to designate an object, nevertheless do not fulfill this condition in special cases.... It is to be demanded that that in a logically perfect language (logical symbolism) every expression constructed as a proper name in a grammatically correct manner out of already introduced [i.e., defined] symbols, in fact designate some object; and that no symbol be introduced as a proper name without assurance that it have a nominatum.

— Gottlob Frege, “On Sense and Nominatum”

Two fundamental questions about language & the two kinds of semantic theories

Two fundamental questions about language: What is the semantic content of each of the sub-sentential components, e.g., singular terms? What contribution does each of the sub-sentential components, e.g., singular terms, make to the semantic content of a proposition?

The two kinds of semantic theories: the naïve theory vs. anti-naïve theories

• The naïve theory: according to the naïve theory, the semantic content of an expression (e.g., singular & general terms) is the object to which it refers.¹ That is, it is committed to the thesis

\[
\text{NT: The meaning of a linguistic expression 'e' is e}
\]

We can then see that, given NT, the meaning of the name ‘Venus’ is the planet Venus, the meaning of the predicate ‘red’ is the colour property being red, the meaning of the relation ‘to the left of’ is the relation being to the left of, etc.

• Anti-naïve theories. These are a family of theories that reject the naïve theory. They might merely call for some degree of modification of the naïve theory or reject it altogether. (∗Note: Mill & Russell are committed to the former; Frege is committed to the latter.) We’ll look a little later at their respective differences.

Four puzzles for the naïve theory of semantic content

According to many philosophers, there (at least) four puzzles that “thoroughly discredit the naïve theory” and hence, motivate some version of anti-naïve theory. These are: (1) Frege’s puzzle about identity statements; (2) the failure of substitutivity in oblique contexts; (3) the problem of true, meaningful negative existentials; (4) the problem of the law of excluded middle.

Frege’s Puzzle (about identity statements)

The problem: if NT is true and the two sentences ‘a = a’ and ‘a = b’ are true, then, since ‘a’ and ‘b’ are extensionally equivalent (i.e., ‘a’ and ‘b’ both refer to the same thing), ‘a’ and ‘b’ cannot differ in meaning. But if so, then we can’t explain the following:

i. ‘a = a’ is an a priori, analytic, necessary truth, one that is trivial and non-informative whereas

¹ The naïve theory is also called the ‘Fido’-Fido theory, the name theory, or the reference theory.
ii. ‘a = b’ is an *a posteriori*, synthetic, contingent truth, one that is non-trivial and informative.

What is more, then we cannot explain how someone could accept a sentence of the form ‘a = a’ but question and even deny a sentence of the form ‘a = b’. Consider Frege’s famous example:

Hesperus, the evening star, is Hesperus,

and

Hesperus is Phosphorus, the morning star.

Some important concepts for understanding Frege’s Puzzle

On the concept of identity statements

It’s important to note that the English word ‘is’ is a multiply ambiguous term. As philosophers have discovered, there are at least five different senses of the word ‘is’: the ‘is’ of existence, the ‘is’ of accidental predication, the ‘is’ of essential predication, the ‘is’ of material constitution, the ‘is’ of identity. Whenever a person uses the ‘is’ of identity, she is asserting an (numerical) identity statement. What is a (numerical) identity statement? A (numerical) identity statement is any statement in which a thing $x$ is said to be numerically one and the same thing as another thing $y$. While such a sentence can be expressed as ‘$x$ is (numerically identical to) $y$’, that is just shorthand for ‘$x$ is numerically one and very same thing as $y$’. Symbolically, identity statements are represented as ‘$x = y$’ where ‘$=$’ is the symbol that designates the identity relation.

An epistemological distinction: *a priori* vs. *a posteriori* (epistemic) justification

Epistemology is that field of philosophical inquiry that is concerned with the nature and scope of human knowledge. As such, epistemologists seek to answer such questions as *Is (human) knowledge possible? and If that knowledge is possible, what is required to have that knowledge and what kinds of things can be known?* To answer such questions, we must inquire into the necessary conditions for a human being to have knowledge. One such condition is the epistemic justification condition. What is the requisite epistemic justification condition? Although philosophers disagree about the details of that condition, they agree that condition recognizes two broad types of epistemic justification: *a priori* epistemic justification and *a posteriori* epistemic justification.

- **A priori epistemic justification**: a person $S$ is epistemically justified *a priori* in believing that a proposition $p$ is true only if (i) $S$ has some justifier $j$ for believing that $p$ is true and (ii) $S$'s having $j$ does not require that $S$ be able to appeal to a particular sensory experience $e$.

Putative examples of propositions that we are epistemically justified *a priori* to believe: that all red things are red, that $a^2 + b^2 = c^2$, etc.

- **A posteriori epistemic justification**: a person $S$ is epistemically justified *a posteriori* in believing that a proposition $p$ is true only if (i) $S$ has some justifier $j$ for believing that $p$ is true and (ii) $S$'s having $j$ does require that $S$ be able to appeal to a particular sensory experience $e$.

Putative examples of propositions that we are epistemically justified *a posteriori* to believe: that there are more than two people in the room, that at least some apples are red, etc.
A linguistic distinction: analytic vs. synthetic truth

Although philosophers continue to debate the nature of truth, following Kant, philosophers nevertheless take it that the truth-conditions for declarative sentences come in two linguistic varieties: analytic and synthetic truths.\(^2\)

- **Analytic truth**: if a declarative sentence \(s\) is analytically true, then (i) \(s\) is true and (ii) \(s\) is made true by virtue of the linguistic form alone.

  Regarding clause (ii) of the analysis of analytic statements. By ‘\(s\) is made true by virtue of the linguistic form alone’, I mean this:

  \[
  \text{AS: } \text{A sentence } s \text{ is made true by virtue of the linguistic form alone iff (a) } s \text{ is made true by virtue of the meaning of the terms alone or (b) the predicate term of } s \text{ is “contained in” the subject term of } s. 
  \]

  AS tells us that there are (a)- and (b)-type analytic statements. Here are examples of both. The statement ‘all bachelors are unmarried males’ is an (a)-type statement given that it is made true by virtue of the meaning of the terms ‘bachelor’ and ‘unmarried male’. The statement ‘all red apples are red’, on the other hand, is not such a statement. That statement is an example of a (b)-type analytic statement that is made true by virtue of the predicate term ‘red’ being “contained in” the subject term ‘red apple’.

- **Synthetic truth**: if a declarative sentence \(s\) is synthetically true, then (i) \(s\) is true and (ii) \(s\) is true in virtue of the predicate concept of \(s\) “extending” the subject concept of \(s\).

  Regarding clause (ii) of the analysis of synthetic statements. By ‘\(s\) is true in virtue of the predicate concept of \(s\) “extending” the subject concept of \(s\)’, I mean this:

  \[
  \text{SS: } \text{A sentence } s \text{ is true in virtue of the predicate concept of } s \text{ “extending” the subject concept of } s \text{ iff } s \text{ gives us information about the concept referred to by the subject term that is not “contained in” that concept, namely, it has the property described by the predicate term.} 
  \]

  For instance, the statements ‘all crows are black’ and ‘water is H\(\text{\textsubscript{2}}\text{O}\)’ are synthetic statements. In both cases, the terms ‘crow’ and ‘water’ don’t “contain” the terms ‘black’ and ‘H\(\text{\textsubscript{2}}\text{O}\)’. We have discovered that ‘crow’ and ‘black’, and ‘water’ and ‘H\(\text{\textsubscript{2}}\text{O}\)’ can be synthesized together so as to form true sentences.

A modal distinction: necessary vs. contingent truths

In addition to the analytic-synthetic distinction regarding truth, the truth-value of declarative sentences is modal: it’s tied to possible worlds. Some sentences have their truth-value necessarily; some sentences have their truth-value contingently.

- **Necessary truth**: a sentence \(s\) is necessarily true iff (i) \(s\) is true and (ii) there is no possible world where \(s\) is false.

---

\(^2\) Although most philosophers accept the analytic-synthetic distinction, in his famous essay “The Two Dogmas of Empiricism,” W.V.O. Quine argues that the distinction is not justifiable.
• **Contingent truth:** a sentence $s$ is contingently true iff (i) $s$ is true and (ii) there is at least one possible world where $s$ is false.

(Note: the same can be stated for falsity. Necessarily false sentences are false in all possible worlds; contingently false sentences are false in some, but not all possible worlds.)

**The failure of substitutivity in oblique/indirect contexts**

The problem: if NT is true and two (or more) expressions $e_1$ and $e_2$ are extensionally equivalent, then if some asserts a sentence $s$ that uses $e_1$, one will be able to substitute $e_2$ for every instance of $e_1$ (and vice-versa) without changing the semantic content of $s$. Yet, this is not possible in oblique/indirect contexts. Therefore, NT is false.

Two examples:

**Propositional attitude contexts: Russell’s Sir Walter Scott Argument**

A. King George IV wanted to know whether or not Sir Walter Scott is the author of *Waverly*.
B. Sir Walter Scott = the author of *Waverly*.

Therefore, C. King George IV wanted to know whether or not Sir Walter Scott is Sir Walter Scott.

Argues Russell, if NT is true, then this argument is valid. But clearly, it’s invalid: while (A) and (B) are true, (C) is false. Regarding (C), Russell famously remarked, “an interest in the law of identity can hardly be attributed to the first gentleman of Europe” (“On Denoting,” *Mind* 14 [1905]). Since it is NT that brings this result, NT is false.³

**Modal contexts: Quine’s number of planets argument**

Given the following law of numerical identity (Leibniz’s Law of the Indiscernability of Identicals)

\[
\text{LL: } \text{If } x = y \& Fx, \text{ then } Fy
\]

where ‘F’ is a predicate variable, the following inference is valid:

D. Necessarily, 9 is an odd number
E. The number of planets = 9

Therefore,
F. Necessarily, the number of planets is odd.

But this inference is clearly invalid. It’s possible for (D) and (E) to be true, but (F) be false. Again, it is NT that brings this result; hence, NT is false.

³ Frege also gives a propositional attitude puzzle using the Hesperus-Phosphorus case:

1. Anaximander believes that that Phosphorus is visible in the morning.
2. Hesperus is Phosphorus
   Therefore,
3. Anaximander believes that that Hesperus is visible in the morning which is invalid, and thus counts against NT.
The problem of true, meaningful negative existentials

The problem: If NT is true, then given the nature of negative existentials, we have a theory that implies absurd results.

On the nature of negative existentials: in order to understand the problem, we need to understand the concept of a negative existential. Let us say that

ES: A statement $s$ is an existential statement iff $s$ is statement of the form ‘$x$ exists’ or ‘$x$ does not exist’.

Any statement of the form ‘$x$ exists’ is a positive existential; any statement of the form ‘$x$ does not exist’ is a negative existential. Consider the following true negative existentials

G: Santa Claus does not exist, and

H: Carnivorous cows do not exist.

Sentences (G) and (H) are (purportedly) of a subject-predicate form (where ‘$x$’ refers to the subject and ‘does not exist’ refers to the predicate). Sentences such as (G) and (H), argue Russell, are problematic since they imply that not only are there things that exist but do not exist, but further, that there are no true, meaningful negative existentials. Consider the following argument he gives.

Russell’s Negative Existentials Argument

i. Sentences (G) and (H) are true, meaningful negative existentials.

ii. Meaningful negative existentials are subject-predicate sentences.

iii. Given NT, a meaningful subject-predicate statement is true iff there is an object (or there are objects) to which the subject expression refers, and this object (or these objects) has (have) the property expressed by the predicate.

Therefore,

iv. Sentences (G) and (H) can be true only if there are objects–Santa Claus and carnivorous cows–to which the subject expressions ‘Santa Claus’ and ‘carnivorous cows’ refers, and these objects have the property of not existing.

v. No objects have the property of not existing. If there are objects to which the subjects of meaningful negative existentials refer, then they exist.

Thus,

vi. Meaningful negative existentials cannot be true.

So,

vii. There are no true, meaningful negative existentials.

Thus,

viii. True, meaningful negative existentials do not exist.

Accordingly,

ix. Sentences (G) and (H) are not true, meaningful negative existentials.
**The problem of the law of excluded middle**

*The problem:* if NT is true, then the law of excluded middle is false. How so? Consider Russell's favorite (and famous) sentence:

I: The present king of France is bald.

Russell thought that statements such as (I) present a *prima facie* counterexample to the law of excluded middle, i.e., the claim that

**LEM:** For any proposition \( p \), either \( p \) or \( \sim p \).

As he says in “On Denoting” (1905):

By the law of excluded middle, either “A is B” or “A is not B” must be true. Hence either “the present King of France is bald” or “the present King of France is not bald” must be true. Yet if we enumerated the things that are bald, and then the things that are not bald, we should not find the present King of France in either list. Hegelians, who love a synthesis, will probably conclude that he wears a wig (p.485).

Russell’s point is that, since there is no present king of France, it looks as though LEM is false. Why? Since there is no present king of France, (I) must be false. But if it is false, then given LEM, we get

I*: The present king of France is not bald.

Yet, as we just said there *is no present king of France!* As such, (I*) must also be false. If both (I) and (I*) are false, then LEM is false. Yet, LEM is arguably necessarily true. Hence, NT is false.\(^4\)

---

\(^4\) © 2018, Richard G. Graziano. All rights reserved. This material may not be used, or duplicated in part or whole without express written permission by the author.